Exotic Particles and Generalized Maxwell theory on Fuzzy Two-Sphere

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Abstract

We consider generalized Maxwell theory and spherical D2-brane. The model is built by introducing a generalized connection put at the origin of two-sphere to describe anyons instead of Chern-Simons term. The energy obtained in this model is very special since the gauge field is dynamic and its energy dominates when the radius of fuzzy two-sphere goes to infinity or if we take large number of charges. Consequently, D2-brane gets high energy. The static potential for two opposite charged exotic particles described by generalized Maxwell theory is found to have screening nature on fuzzy two-sphere instead of confinement which is a special property of the system on the plane.

1 Introduction

Various brane configurations have attracted much attention over the recent years and several papers have been devoted to the study of a relationship between noncommutative geometry [1] and string theory [2] and the relationship between D-branes with different dimensions as well [3, 4]. The appearance of noncommutative geometry in string theory can be understood from a different point of view. For example, a D2-brane can be constructed from multiple D0-branes by imposing a noncommutative relation on their coordinates in matrix theory or under the strong magnetic field the world volume coordinates of a D2-brane become noncommutative by considering the quantum Hall system [4] and the magnetic field charge is interpreted as the number of D0-branes.

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In this work, we consider exotic particles described by generalized Maxwell theory in which we introduce a generalized connection in fuzzy two-sphere which has a dual description in terms of an abelian gauge field on a spherical D2-brane and is interpreted as a bound state of a spherical D2-brane and D0-branes. The exotic particles are known as excitations and quasi-particles or anyons; i.e. fermions (bosons) carrying odd (even) number of elementary magnetic flux quanta [5]. They are living in two-dimensional space as composite particles having arbitrary spin, and they are characterized by fractional statistics which are interpolating between bosonic statistics and fermionic one [5, 6]. One of the field-theories describing anyons is the model, where the matter is interacting with the Chern-Simons (CS) gauge field [7]. In the reference [8], Stern has introduced another approach to treat anyons that does not require the CS term, but introduces a generalized connection with which the conserved U(1) current is coupled in a gauge invariant way [9]. In this model the gauge field is dynamic and the potential has the confining nature which makes the model different [10].

This paper is devoted to treat the same system but on two-sphere. Among the main results in this work is the change of the potential's nature; there is no confinement nature any more, and the disappearance of the confinement in two-sphere case for the exotic system is very interesting result. It was shown in [11] that compact Maxwell theory in (2+1)-dimensions confines permanently electric test charges and the usual two-dimensional Coulomb potential is $V(R) \sim lnR$. Since the electrostatic potential has the form $V(R) \sim R$ and holds for all values of the gauge coupling, the compact (2+1)-dimensional Maxwell theory does not exhibit any phase transition, i.e., the confinement is permanent. In the present paper, the things are changed by treating the exotic system in high dimensions and $V(R) \sim \frac{1}{R}$ with R is the distance between two opposite charged exotic particles. Another important result we get is at the level of energy; D2-brane gets high energy if the radius r of the two-sphere goes to infinity and it is higher if the number of charges is large which makes the system very special.

The main results of this work: The fuzzy two-sphere is realized as one of D2-brane descriptions with special properties because of the generalized Maxwell theory. Among these properties we get the energy of gauge field dominates when the radius of fuzzy two-sphere goes to infinity, then the energy of flat D2-brane which is a dual of fuzzy two-sphere becomes high which is different from the case of quantum Hall effect (QHE) where the energy of flat D2-brane goes to zero. An important remark is that our system could be identified to QHE in high dimensions only if the radius r and the number of charges N go to zero. Also, what makes the model very different and very special is that the potential loses the confining nature in fuzzy two-sphere case.

2   Generalized Connection and Anyons

The simplest way to realize fractional statistics characterizing anyons in three-dimensional space-time is usually by adding a Chern-Simons term to the action. Recently, a novel way was introduced in [8] to describe anyons without a Chern-Simons term. Thus, a generalized connection was considered in (2+1)-dimensions denoted A_μ^θ, $\mu = 0, 1, 2$. The gauge theory is defined by the following Lagrangian

$$L_\theta = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu^\theta \tag{1}$$

with $A_\mu^\theta \equiv A_\mu + \frac{\theta}{2}\epsilon_{\mu\nu\rho}F^{\nu\rho}$ and θ is real parameter in Minkowski space. The Lagrangian L_θ describes Maxwell theory that couples to the current via the generalized connection rather than the usual one. This coupling is gauge invariant as long as J^μ is a conserved external current. In this theory, the gauge fields are dynamic and the canonical moments are $\pi^\mu = F^{\mu 0} + \theta\epsilon^{0\mu\nu}J_\nu$ which results in the usual primary constraint $\pi^0 = 0$ and $\pi^i = F^{i0} + \theta\epsilon^{0ij}J_j$ ($i, j = 1, 2$). Thus the magnetic field is $B = \epsilon_{ij}\partial^i A^j$ and the electric field is $E^i = \pi^i - \theta\epsilon^{ij}J_j$.

Now, accordingly to (1), the equations of motion for A_μ give

$$\partial^\nu\partial_\nu A_\mu = J_\mu + \theta\epsilon_{\mu\nu\rho}\partial^\nu J^\rho. \tag{2}$$

Then, we consider the simplest case of a static pointlike particle located at the origin which is described by $J^0 = e\delta^{(2)}(x)$. By solving (2) for the gauge field one finds

$$A_0 = \frac{lnr}{2\pi}, \qquad A_1 = \frac{\theta x_2}{2\pi r^2}, \qquad A_2 = \frac{\theta x_1}{2\pi r^2},$$

with $r^2 = x_1^2 + x_2^2$. This background describes one unit of an electrically charged particle and an infinitely thin magnetic flux with total flux θ both located at the origin and the shift in the statistics of the particle is fixed by the Aharonov-Bohm effect to be

$$\Delta\phi = \theta. \tag{3}$$

We note that in the case of Chern-Simons theory, the phase is two times θ and this is due to the fact that the charged particle is winding around a magnetic flux while in the present theory we also have the contribution of a flux tube winding around the charged particle. Another reason is that with A_μ^θ construction a long range electric field is also generated which couples to the current and gives exactly the same phase.

For a static charged particle located at the origin and $J^i = 0$, the static electromagnetic fields are

$$\begin{aligned} B(x) &= e\theta\delta^{(2)}(x) \\ E_i(x) &= -\frac{e}{2\pi}\frac{x_i}{r^2} \end{aligned} \tag{4}$$

and the total magnetic flux attached to N charged particles is

$$\Phi = \int_V d^2x B(x) = e\theta N. \tag{5}$$

We note that the both L_{CS} (the lagrangian in Chern-Simons theory) and L_θ lead to fractional statistics by the same mechanism of attaching a magnetic flux to the charged particles but the physics they describe is quite different. We remark, for example, that in this theory the interaction potential is an object of considerable interest [10]. The potential has confining nature; i. e., it grows to infinity when the natural separation of the physical degrees of freedom grow, but in the Maxwell-Chern-Simons theory, the Chern-Simons term turns the electric and magnetic fields massive leading to a screening potential between static charges.

3 Anyons and Fuzzy Two-Sphere

Now, let us consider exotic particle moving on a two-sphere instead of a plane in the background of a monopole put at the origin. First, the two-sphere is $S^2 \sim CP^1 = \frac{SU(2)}{U(1)}$ and the representations of $SU(2)$ are given by the standard angular momentum theory.

The coordinates of fuzzy two-sphere are given by the $SU(2)$ algebra

$$[X_i, X_j] = i\alpha\epsilon_{ijk}X_k, \quad X_i = \alpha L_i, \tag{6}$$

L_i is the total angular momentum with the representation to be the spin ℓ and α is a dimensionful constant. We note that around the north pole of S^2 labeled by $L_3 = \ell$, the fuzzy two-sphere algebra becomes a noncommutative plane if $\ell \to \infty$,

$$[X_i, X_j] = i\alpha^2\ell\epsilon_{ij}I, \tag{7}$$

with I is the identity.

3.1 Connection

To construct the connection which goes to the generalized connection given above when the radius of fuzzy two-sphere goes to infinity we use the first Hopf map as known in the literature which is a map from S^3 to S^2 and naturally introduces a $U(1)$ bundle on S^2. Then, the two-sphere can be parameterized by two complex coordinates u_α such that $u_\alpha^\star u_\alpha = 1$ with $u_\alpha \sim e^{i\theta}u_\alpha$. A spatial coordinate x_i on S^2 with radius r is written in terms of u_α's as

$$x_i = ru^\dagger\sigma_i u \tag{8}$$

with σ are Pauli matrices. The vector potential on S^2 is

$$A_i dx_i = -i\gamma u_\alpha^\star du_\alpha, \tag{9}$$

with γ is integer due to the Dirac quantization rule and $u_\alpha^\star$ is the complex conjugate of u_α.

Thus, the Hopf spinor satisfying (8) is given by

$$u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \frac{1}{\sqrt{2r(r+x_3)}}\begin{pmatrix} r+x_3 \\ x_1+ix_2 \end{pmatrix}e^{i\chi}, \tag{10}$$

$e^{i\chi}$ is a $U(1)$ phase. The connection is defined as

$$A_i dx_i = -i\frac{\hbar}{e}u_\alpha^\star du_\alpha = \frac{\hbar}{2er(r+x_3)}\epsilon_{ij3}x_j dx_i. \tag{11}$$

By leading with the motion of an exotic particle (charged particle-magnetic flux composite) on two-sphere, the monopole charge is $\frac{\hbar}{2e} = \frac{\theta}{4\pi}$ which is identified with the connection in two dimensional space for $r \to \infty$ discussed in section 2. By generalizing the spinor to $(2S+1)$-components spinor $u_{(S)}$, the monopole charge becomes $\frac{\theta}{4\pi} = \frac{\hbar S}{e}$ and $x_i = \frac{1}{S}ru_{(S)}^\dagger\sigma_{(S)_i}u_{(S)}$, $x_ix_i = r^2$, where $\sigma_{(S)_i}$ is the spin S representation of $SU(2)$.

Now, for simplicity we consider a static particle at $\mathbf{x}'$. The magnetic and electric fields are given in (4) and the charge-magnetic dipole is defined by the current

$$J_0 = e\delta^{(3)}(\mathbf{x}-\mathbf{x}') \quad J_i = \frac{\phi}{e}\epsilon_{im}\partial_m J_0, \tag{12}$$

ϕ is the dipole's moment.

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3.2   Generalized Maxwell Theory

The Hamiltonian of this system is written as follows

$$H = \frac{1}{2mr^2}M_iM_i + \int d^3x(-\frac{1}{2}F_{i0}F^{i0} + \frac{1}{4}F_{ij}F^{ij} - \frac{\theta}{2}\epsilon_{ij}J^0F^{ij}) \tag{13}$$

such that for a static point like particle $J_i = 0$ and the primary constraint is

$$\pi^0 = 0$$

which leads to the secondary constraint

$$\partial_i\pi^i - J^0 = 0$$

with π^μ is the canonical momentum of gauge field A^μ. M_i is the orbital angular momentum of the charged particle

$$\begin{aligned} M_i &= \epsilon_{ijk}x_j(-i\hbar\partial_k + eA_k^\theta) \\ &= \epsilon_{ijk}x_j(-i\hbar\partial_k + eA_k + \tfrac{e\theta}{2}\epsilon_{knm}F^{nm} + \tfrac{e\theta}{2}\epsilon_{kn0}F^{n0}) \end{aligned} \tag{14}$$

where $i,j,k,n,m = 1,2,3$ and A_k^θ is the generalized connection. The strength field $F^{\mu\nu}$ is

$$\begin{aligned} F^{nm} &= -\tfrac{\theta}{4\pi}\epsilon_{nml}\tfrac{x_l}{r^3} \\ F^{n0} &= \tfrac{\theta}{4\pi r(r+x_3)}\epsilon_{nl3}(\dot{x}_l - \dot{r}x_l\tfrac{2r+x_3}{r(r+x_3)}), \end{aligned} \tag{15}$$

we note that $\epsilon_{kn0}F^{n0} = 0$ since $\epsilon_{nl3}\epsilon_{kn0} = 0$ because $l \neq 0$. Then

$$M_i = \epsilon_{ijk}x_j(-i\hbar\partial_k + eA_k - \frac{e\theta^2}{4\pi}\frac{x_k}{r^3}). \tag{16}$$

Thus the Hamiltonian (13) of this system is reduced to

$$H = \frac{1}{2mr^2}M_iM_i + \frac{1}{2}\int d^3x(E_i^2 + B^2), \tag{17}$$

with $\epsilon_{oij}J^0F^{ij} = \epsilon_{oij}J^0\tfrac{-\theta}{4\pi}\epsilon^{ijk}\tfrac{x_k}{r^3} = 2\tfrac{-\theta}{4\pi}\delta_{0k}J^0\tfrac{x_k}{r^3} = 0$ in (13) since $k = 1,2,3 \neq 0$. Accordingly to (4,5), we calculate the second term of H in three-dimensional space and the Hamiltonian is

$$H = H_0 + e^2\theta^2N + \frac{e^2r}{3\pi}, \tag{18}$$

with

$$H_0 = \frac{1}{2mr^2}M_iM_i. \tag{19}$$

The remark we get from this subsection is that the Hamiltonian is different from the one describing QHE and they are identified ($H \sim H_0$) only if $N, r \longrightarrow 0$.

3.3 Realization of Fuzzy Two-Sphere

First, we remark that the orbital angular momentum of the particle M_i given by (14) satisfy the following deformed commutation relations

$$[M_i, M_j] = i\hbar\epsilon_{ijk}(M_k + \frac{e\theta}{2\pi r}x_k). \tag{20}$$

This means that the total angular momentum generalizing the $SU(2)$ algebra should be defined as

$$L_i = M_i - \frac{e\theta}{2\pi r}x_i \tag{21}$$

and we get

$$[L_i, L_j] = i\hbar\epsilon_{ijk}L_k$$

$$[L_i, M_j] = i\hbar\epsilon_{ijk}M_k \tag{22}$$

$$[L_i, x_j] = i\hbar\epsilon_{ijk}x_k.$$

Consequently, by simple calculation we find that

$$[L_i, H] = 0,$$

then $SU(2)$ symmetry is generated by L_i. We also see that

$$\begin{aligned} M_iM_i &= L_iL_i - (\tfrac{e\theta}{2\pi})^2 \\ &= \hbar^2(l(l+1) - 4S^2), \end{aligned} \tag{23}$$

with ℓ is the eigenvalue of L_3. In what follows we suggest that $\ell = n+2S$ with $n = 0, 1, 2....$

The noncommutative Geometry known as fuzzy two-sphere is described by the guiding center coordinates since the exotic particle obeys the cyclotron motion as well-known in the planar system. These coordinates are defined as

$$X_i = \frac{2\pi r}{e\theta}L_i, \tag{24}$$

then they are related to the commutative coordinates by

$$X_i = \frac{2\pi r}{e\theta}M_i - x_i. \tag{25}$$

They satisfy the following commutative relations

$$[X_i, X_j] = i\hbar\epsilon_{ijk}\frac{2\pi r}{e\theta}X_k, \tag{26}$$

and the fuzzy two-sphere is satisfied for the motion of exotic particle on two-sphere. Its radius is given by the quadratic Casimir of $SU(2)$

$$r^2 = \hbar^2(\frac{2\pi r}{e\theta})^2 2S(2S + 1). \tag{27}$$

According to (23,24), we get the radius of the cyclotron motion in the n-th level

$$r_n^c = \frac{2\pi r}{e\theta}\hbar\sqrt{2S(2n+1)+n(n+1)}. \tag{28}$$

For the lowest level we get

$$r_0^c = \frac{r}{\sqrt{2S}}, \tag{29}$$

which is identified with the one obtained in the lowest Landau Level discussed in [4]. Also we remark that r_0^c is much smaller that r in the strong magnetic field limit, and x_i are identified with X_i.

3.4 Energy in Fuzzy Two-Sphere Case

Owing to (23), the energy eigenvalue of H (18) is

$$E_n = \frac{\hbar^2}{2mr^2}(2S(2n+1)+n(n+1)) + e^2\theta^2 N + \frac{e^2 r}{3\pi}. \tag{30}$$

Then we notice that this model could be identified with the one treated in the references [4] only in the following case: If both of the radius of fuzzy two-sphere and the number of charges N are too small; i.e. $r, N \longrightarrow 0$. Also, we note that n in (30) indicates the level index which could be identified with the Landau level index in the Chern-Simons theory only for small r and N. The variance energy between the lowest level $n = 0$ and the first level is

$$\Delta E = \frac{\hbar^2(2S+1)}{mr^2}.$$

As remark, the lowest level is also realized in our system which is identified to the Lowest Landau level phenomena if the number of charges is too small and $r \longrightarrow 0$ with $\frac{S}{mr} \gg 1$ and the energy induced by the dynamic gauge field is ignored. Otherwise, if the above case is not satisfied; i.e. $r \gg 1$ or $N \gg 1$, the model is now totally different. Thus the variance energy of the system is

$$\Delta E = 0,$$

and the energy is dominated by the one of the two last terms of (28) or both. Then the energy is

$$E_n = e^2\theta^2 N + \frac{e^2 r}{3\pi}, \quad \forall n. \tag{31}$$

We notice here that the variance energy of the system will depend only on the variance of the number of particles or the radius.

Consequently, dealing with the case of exotic particles system in which we introduce a generalized connection put at the origin of two-sphere we get a noncommutative geometry. The energy obtained in this model is very special and too different from the one obtained in QHE case since the gauge field in this system is dynamic. We notice that the energy of gauge field dominates when the radius of fuzzy two-sphere goes to infinity; i.e. the flat D2-brane which is a dual of fuzzy two-sphere has high energy. We remark that this result is definitely different from the one could be obtained in the case of QHE. In this latter case if $r \longrightarrow \infty$ the fuzzy two-sphere goes to flat D2-brane having low energy which goes to zero.

3.5 Potential

We complete this section by giving another interesting remark. As known, the potential has confining nature in two-dimensional space when the generalized connection is introduced instead of adding CS-term; i. e., the potential grows to infinity when the natural separation of the physical degrees of freedom grows.

After giving the energy we may now proceed to discuss the interaction energy between pointlike sources in the model under consideration. This can be done by computing the expectation value of the energy operator H in a physical state $| \Omega \rangle$ by following the mechanism used in [12]. We consider the stringy gauge-invariant $| \bar{\Psi}(y)\Psi(r) \rangle$ state,

$$| \Omega \rangle \equiv | \bar{\Psi}(y)\Psi(y') \rangle = | \bar{\psi}(y)e^{-ie\int_y^{y'} dz^i A_i(z)} \psi(y') | 0 \rangle, \tag{32}$$

where $| 0 \rangle$ is the physical vacuum state and the integral is to be over the linear spacelike path starting at y and ending at y', on a fixed time slice. Note that the strings between exotic particles have been introduced to have a gauge-invariant state $| \Omega \rangle$, in other terms, this means that the elementary particles (bosons or fermions) are now dressed by a cloud of gauge fields.

From the foregoing Hamiltonian discussion, we first note that

$$\pi_i | \bar{\Psi}(y)\Psi(y') \rangle = \bar{\Psi}(y)\Psi(y')\pi_i | 0 \rangle + e \int_y^{y'} dz_i \delta^3(x - z) | \bar{\Psi}(y)\Psi(y') \rangle. \tag{33}$$

Owing to (17,31) and the fact that we consider a static pointlike particle; so $\pi_i = F_{0i} = E_i$, we get the expectation value of the Hamiltonian as

$$\langle \Omega | H | \Omega \rangle = \langle 0 | H | 0 \rangle + \frac{e^2}{2} \int d^3x \Big(\int_y^{y'} dz_i \delta^3(x - z) \Big)^2, \tag{34}$$

with x and z are three-dimensional vectors. Remembering that the integrals over z_i are zero except on the contour of integrations.

The last term of (34) is nothing but the Coulomb interaction plus an infinite self-energy term. In order to carry out this calculation we write the path as $z = y + \alpha(y - y')$ where α is the parameter describing the contour. By using the spherical coordinates the integral under square becomes

$$\int_y^{y'} dz_i \delta^3(x - z) = \frac{y - y'}{|y - y'|^2} \int_0^1 d\alpha \frac{1}{\alpha} \delta(|y - x|, \alpha|y' - y|) \sum_{\ell,m} Y_{\ell m}^*(\theta', \phi') Y_{\ell m}(\theta, \phi). \tag{35}$$

Using the usual properties for the spherical harmonics and after subtracting the self-energy term, we obtain the potential as

$$V = -\frac{e^2}{4\pi} \frac{1}{|y' - y|} \tag{36}$$

This result lets us to draw attention to the fact that with fuzzy two-sphere the generalized Maxwell theory doesn't have confining nature any more which was a special property for anyons described by generalized Maxwell theory in two-dimensional space. Thus the problem of confinement could be solved by considering the two-sphere in stead of two-dimensions.

4 Conclusion

In this paper, we have used the generalized Maxwell theory on two-sphere instead of two-dimensional space. This leaded to get some results totally different from those gotten in the case of QHE in high dimensions [4]. By considering the exotic particles described by generalized Maxwell theory, the energy produced by the gauge field is involved in the energy of the system (18) depending on the number of charges and the radius of the sphere. We remark that the energy of gauge field dominates when the radius of two-sphere goes to infinity; i. e. the energy of flat D2-brane is generated by the gauge field leading to high energy. We also notice that the energy becomes more higher if the number of charges is large. Another important remark is that with fuzzy two-sphere the static potential for two opposite charged exotic particles loses automatically its confining nature without adding the CS-term; i. e. by plunging the generalized Maxwell theory in high dimensions the potential has screening nature.

Acknowledgments

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